Development of Multi-scale and Multi-physics Finite Element Analysis for Integrated Functional Evaluation of Articular Cartilage

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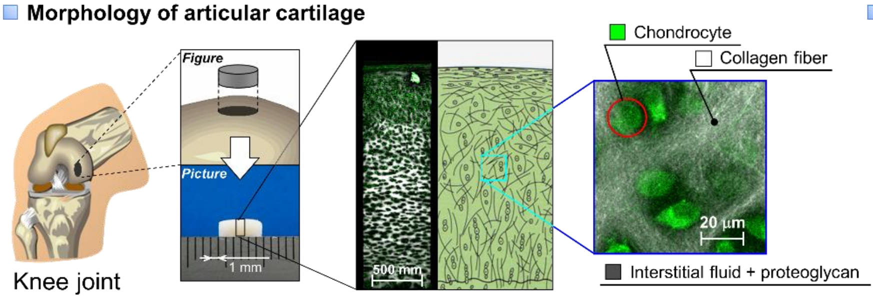
1. Back ground of study

Morphology of articular cartilage



K. FUJII and H. INOUE, Biology of Bone and Cartilage -Application of Basic Science for Practice Medicine-, (2002), p.85, KANEHARA & Co., Ltd.

Material of articular cartilage

- Solid phase
  - Chondrocyte
  - Collagen fiber
  - Proteoglycan(PG)
- Liquid phase
  - Interstitial fluid

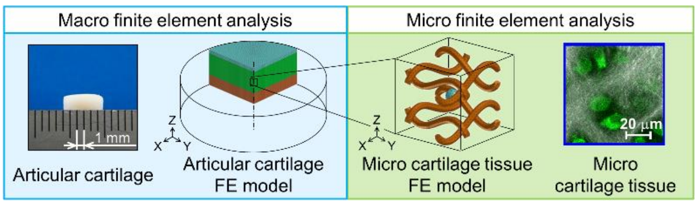
- Hierarchical structure
- Biphasic property

<Objective>

Development of multi-scale and multi-physics finite element analysis to evaluate stress on chondrocyte in articular cartilage.

2. Multi-scale multi-physics finite element method

Multi-scale finite element method

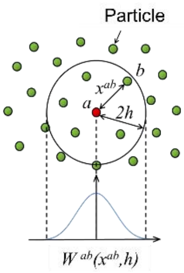


- Macro FE model
  Articular cartilage disk specimen
- Micro FE model
  Chondrocyte, collagen fiber, hydro-gel(PG + Interstitial fluid)

FE nodal displacements → Assign to boundary of micro FE model

Multi-physics finite element method

- Smoothed Particle Hydrodynamics(SPH) method



<SPH continuity equation>

$$\frac{d\rho^a}{dt} = \rho^a \sum_{b=1}^{N} \frac{m^b}{\rho^b}(u_i^a - u_i^b)\frac{\partial W^{ab}}{\partial x_i}$$

<SPH momentum equation>

$$\frac{Du_i^a}{Dt} = \sum_{b=1}^{N} m^b\left(\frac{p^a}{\rho^{a2}}+\frac{p^b}{\rho^{b2}}+\Pi^{ab}\right)\frac{\partial W^{ab}}{\partial x_j}+F_i$$

Artificial viscosity

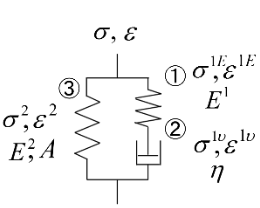
$$\Pi^{ab} = \begin{cases} \dfrac{-\alpha\bar{c}^{ab}\phi^{ab}+\beta\phi^{ab2}}{\bar{\rho}^{ab}} & u^{ab}\cdot x^{ab}<0 \\ 0 & u^{ab}\cdot x^{ab}\geq 0 \end{cases}$$

h : Smoothing length
x^a : Position of particle a
x^{ab} : x^b-x^a
ρ^a : Density of particle a
$\bar{\rho}^{ab}$: Average density of particle a and b
m^a : Mass of particle a
u^a : Velocity of particle a
p^a : Pressure of particle a
F_i : External force
$\bar{c}^{ab}$: Average sound velocity of particle a and b
φ^{ab} : $h\cdot u^{ab}\cdot x^{ab}/((x^{ab})^2+(0.1h)^2)$

3. Multi-scale multi-physics finite element analysis of articular cartilage

PAM-CRASH & User's Material Subroutine and SPH(Murnaghan Equation of State for Slid Element and SPH), ESI Ltd.

Material properties of articular cartilage



Nonlinear viscoelastic (NVE) model

① Linear elastic element

$$\sigma^{1E} = E^1\varepsilon^{1E}$$

② Damper element

$$\sigma^{1v} = \frac{2\eta}{3}\dot{\varepsilon}^{1v}$$

③ Nonlinear elastic element

$$\sigma^2 = E^2 A\log_e\{\cosh(\bar{\varepsilon}/A)\}\varepsilon^2$$

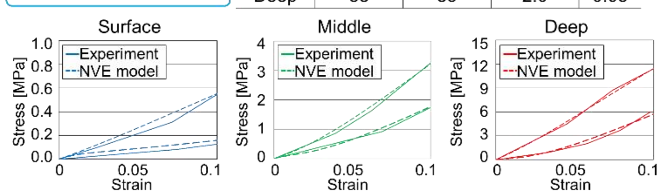
$$\dot{\sigma}_{ij} = (D^1_{ijkl}+D^3_{ijkl})\dot{\varepsilon}_{kl} - \frac{3}{2\eta}D^1_{ijkl}\sigma_{kl} + \frac{3}{2\eta}D^1_{ijkl}D^2_{klmn}\varepsilon_{mn}$$

E^1 : Linear elastic modulus
η : Viscosity coefficient
D^1 : Linear elastic tensor
E^2 : Nonlinear elastic modulus
A : Nonlinear coefficient
D^2, D^3 : Nonlinear elastic tensor

Curve fitting by least square approximation method

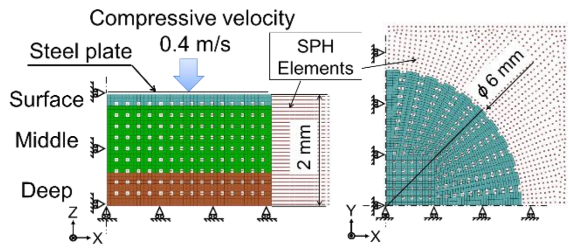
NVE material properties

	E^1 [MPa]	E^2 [MPa]	η [MPa·s]	A
Surface	5	2	1.2	0.05
Middle	17	25	2.1	0.05
Deep	88	85	2.0	0.05



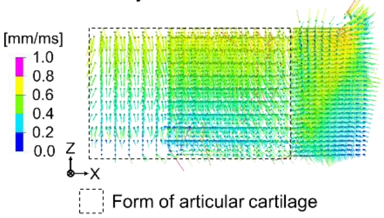
Macroscopic : Articular cartilage

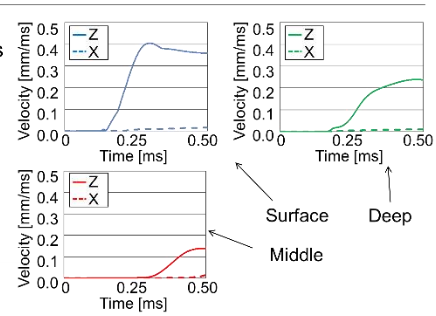
- Macro-FE model



- Results of macro finite element analysis

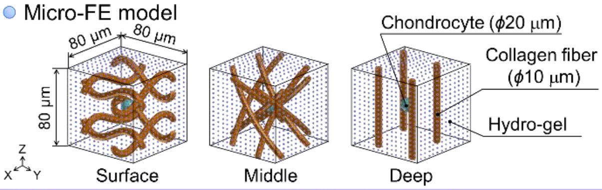
<Flow velocity>



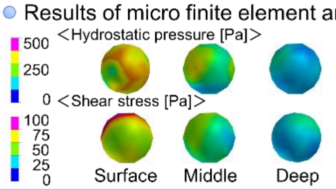


Microscopic : Chondrocyte, collagen fiber, hydro-gel

- Micro-FE model



- Results of micro finite element analysis



Average values of hydrostatic pressure and shear stress

Layer	Surface	Middle	Deep
Hydrostatic pressure [Pa]	213	116	45
Shear stress [Pa]	61	40	16

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